AN INTRODUCTION TO DIFFERENTIATION

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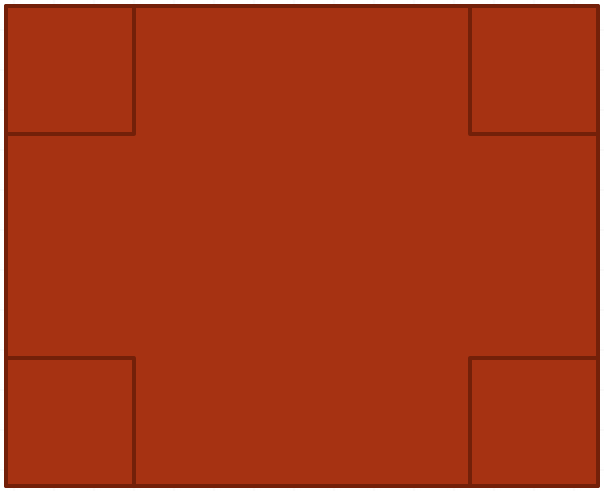
At the end of this presentation you will be able to solve the following question:

- An open box is made from a square sheet of cardboard, with sides half a metre long, by cutting out a square from each corner, folding up the sides and joining the cut edges. Find the maximum capacity of the box.



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TOOLS NEEDED

- Differentiation
- A good knowledge of functions

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OUTLINE

- What is differentiation
- Notations
- Differentiation of:
  - constants and multiples of x
  - Products & quotients
  - composite functions (function of a function)
- Stationary points
- Table of differentiation

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WHAT IS DIFFERENTIATION

A process of finding the general expression for the gradient of a curve at any point.

A straight line is also considered a curve

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NOTATION

- The term $\frac{d}{dx}$ means “the derivative with respect to x”
- $\frac{dy}{dx}$ means “the derivative of y with respect to x”
- $\frac{d}{dx}(x^2 - x)$ means “the derivative of $(x^2 - x)$ with respect to x”
- $\frac{dy}{dx}$ is also written as $f'(x)$ which means “f prime of x”
- $\frac{d^2y}{dx^2}$ is also written as $f''(x)$ which is the derivative of $f'(x)$

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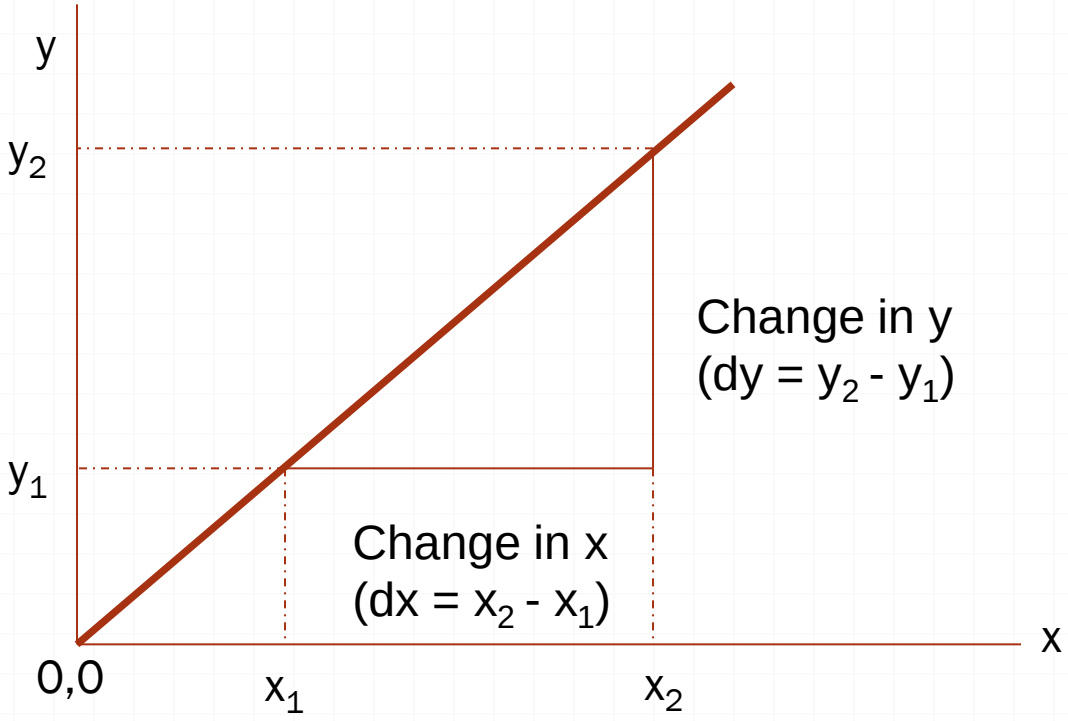

- The gradient of this line is constant and is equal to (dy/dx)
- Hence the differentiation of this line gives us (dy/dx)
- Note : differentiation is the process, the derivative is the end product

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DIFFERENTIATION OF CONSTANT MULTIPLES OF X

The following rule is used, where a is a constant multiple of x^n

$$\frac{d}{dx} ax^n = a \times nxn^{-1}$$

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DIFFERENTIATION OF PRODUCTS OF FUNCTIONS

The following rule is used, where u and v are functions of x

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

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DIFFERENTIATION OF QUOTIENTS OF FUNCTIONS

The following rule is used, where u and v are functions of x

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

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DIFFERENTIATION OF COMPOSITE FUNCTIONS

The chain rule : $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ is used to differentiate composite functions.

Where y is a function of u and u is a function of x, as in

Y = f(g(x)) {y is equal to f of g of x}
U = g(x) {u is equal to g of x}

That is, y = f(u) {y is equal to f of u}

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LOCATING STATIONARY POINTS

At stationary points $\frac{dy}{dx} = 0$

To locate stationary points,

- We find the first derivative of x ($\frac{dy}{dx}$)
- We equate it to 0
- we solve for x in the equation
- Then we substitute our values of x into our original expression to find our corresponding values for y

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TO DETERMINE WHICH STATIONARY POINTS ARE MAXIMUM OR MINIMUM

Stationary points are maximum when

$$\frac{d^2y}{dx^2} < 0$$

And are minimum when

$$\frac{d^2y}{dx^2} > 0$$

Other methods which are more complicated may sometimes be required

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